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ANALYSIS AND MODIFICATION OF CIP METHOD FOR HYPERBOLIC EQUATIONS

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Abstract

The CIP method has been developed to solve the hyperbolic equations, and applied for calculations of various fluid flows. However, there has been little of necessary discussion about the method itself. In the present paper, the consistency and accuracy of the CIP method are investigated, and it is indicated that this method may permit wrong solutions. In order to avoid such wrong solutions, a certain type of artificial viscosity is introduced for the purpose of obtaining high resolution on discontinuities and suppressing numerical oscillations. A procedure that modifies the original scheme is presented to preserve monotonicity, and several monotonicity preserving CIP schemes are proposed. Numerical examples validate this analysis and demonstrate the superior performance of the artificial viscosity terms introduced and the schemes proposed.

1 INTRODUCTION

The CIP method (cubic-interpolated pseudo-particle method) was developed by Yabe *et al.* [1-4] to solve the hyperbolic equations. The procedure of this method is simple and can be straightforwardly extended to multidimensional cases. The CIP method has been successfully applied for calculations of compressible flows, incompressible flows, two-phase fluids, heat conduction, *etc.*, and these numerical results show that the method is stable and less diffusive, for example, see [3-6]. However, the construction of the method is quite different from that of most modern schemes, and, as indicated by the authors of the method, the reason why the CIP method works well is not clear yet. For better comprehension of the method as well as its further development, the authors has studied the CIP method [7-8]. In the current paper, we discuss the consistency, accuracy, wrong solutions and artificial viscosity of this method, and then proceed to present several monotonicity preserving the CIP schemes.

2 OUTLINE OF THE CIP METHOD

Consider the following scalar hyperbolic equation

$$v_t + (fv)_x = g, \tag{1}$$

where f and g are functions of v. It is derived from (1) that

$$V_t + fV_x = H, \tag{2}$$

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in which $V = (v, v_x)^T$, $H = (G, G_x - v_x f_x)^T$, $G = g - v f_x$. Equation (2) can be splitted into the following two equations

$$V_t = H, \tag{3}$$

$$V_t + f V_x = 0. \tag{4}$$

For (3), Yabe and Aoki [3] gives the operator L_1:

$$w_i^* = w_i^n + G_i^n \Delta t, \tag{5}$$

$$w_{x_i}^* = w_{x_i}^n + \frac{w_{i+1}^* - w_{i-1}^* - w_{i+1}^n + w_{i-1}^n}{2\Delta x} - \frac{\Delta t}{2\Delta x} w_{x_i}^n (f_{i+1}^n - f_{i-1}^n). \tag{6}$$

where w_i^n and $w_{x_i}^n$ are numerical solutions at time t^n and grid node x_i, Δt and Δx are respectively time step and grid spacing. The feature of the CIP method is that equation (4) is solved by the CIP schemes. The CIP0 scheme, being the basic form of the CIP schemes, is a shift operator L_2:

$$w_i^{n+1} = P(x_i - f_i^n \Delta t), \tag{7}$$

$$w_{x_i}^{n+1} = P_x(x_i - f_i^n \Delta t), \tag{8}$$

where $P(x)$ is the piecewise cubic Hermite interpolation given by w_i^* and $w_{x_i}^*$. If $f_i^n < 0$, $P(x)$ reads

$$\begin{aligned} P(x) &= ((a_i X + b_i) X + w_{x_i}^*) X + w_i^*, \\ a_i &= \frac{2(w_i^* - w_{i+1}^*)}{\Delta x^3} + \frac{w_{x_i}^* + w_{x_{i+1}}^*}{\Delta x^2}, \\ b_i &= \frac{3(w_{i+1}^* - w_i^*)}{\Delta x^2} - \frac{2 w_{x_i}^* + w_{x_{i+1}}^*}{\Delta x}. \end{aligned} \tag{9}$$

Here, $X = x - x_i$, $x \in [x_i, x_{i+1}]$. If $f_i^n > 0$, $P(x)$ is determined in the similar way. Combining (7), (8), and (9) gives rise to

$$w_i^{n+1} = ((a_i \xi + b_i)\xi + w_{x_i}^n)\xi + w_i^*, \tag{10}$$

$$w_{x_i}^{n+1} = (3 a_i \xi + 2 b_i)\xi + w_{x_i}^*, \tag{11}$$

where $\xi = -f_i^n \Delta t$. The whole procedure of the CIP method is then [3]

$$W_i^{n+1} = L_2 L_1 W_i^n. \tag{12}$$

Here W is the numerical approximation to V. An alternative form associated with of (12) can thus be written as

$$W_i^{n+1} = L_1 L_2 W_i^n. \tag{13}$$

If (1) is linear and homogeneous: $f = const, g = 0$, (12) and (13) will become (7) and (8). For more details about the CIP method, *c.f.* [3] and [4].

3 CONSISTENCY AND ACCURACY

When w_i and w_{xi} are, respectively, v_i and v_{xi}, the exact solution, the error of cubical Hermite interpolation (9) reads

$$| v_{px} - P_{px} | \le \frac{m}{(4-p)!} \Delta x^{4-p}, \quad 0 \le p \le 4, \tag{14}$$

where $v_{px} = \partial^p v / \partial x^p$, $P_{px} = \partial^p P(x) / \partial x^p$, $m = max \mid v_{5x} \mid$. Equation (14) indicates that P_{px} given by (9) is of $(4-p)$th order accuracy, thus P and P_x are of fourth and third order accuracy. It can be proven that, as $\Delta x \to 0$, P_{px} given by (9) is stable and it converges uniformly to v_{px} [9].

Let v be smooth and satisfy (3). By introducing Taylor series to (7) and (8) one has

$$v_t + fv_x = -\Delta t f f_v {v_x}^2 + O(\Delta t^2, \Delta t^2\Delta x, \Delta t\Delta x^2), \tag{15}$$

and

$$v_{xt} + fv_{xx} = -\Delta t f f_v v_x v_{xx} + O(\Delta t^2, \Delta t\Delta x, \Delta x^2). \tag{16}$$

Therefore, equations (7) and (8), the CIP0 scheme, are consistent to (3) and (4), respectively, and their accuracy is the first-order. Since v_{xx} and v_{xxx} are not involved in the first terms of RHS of (15) and (16), respectively, the CIP0 scheme does not introduce much diffusion to numerical solutions. It is also known from (7) and (8) that $w_i^{n+1} = w_i^*$ and $w_{x_i}^{n+1} = w_{x_i}^*$ at grid nodes where $f_i^n = 0$, i.e., the CIP0 scheme has no numerical viscosity at all.

Let v be a smooth solution of (2) and put it into (13). Straightforward calculations yield:

$$v_t + fv_x = \frac{\Delta t}{2}\left(Gf_x - 2ff_xv_x - \frac{GG_x}{v_x}\right) + O(\Delta t^2, \Delta t^2\Delta x, \Delta t\Delta x^2), \tag{17}$$

$$\begin{aligned} v_{xt} + fv_{xx} = \frac{\Delta t}{2}\Big(&f_xG_x + f_{xx}G - 2ff_xv_{xx} - \frac{{G_x}^2}{v_x} - \frac{GG_{xx}}{v_x} - \frac{GG_xv_{xx}}{{v_x}^2}\Big) \\ &+O(\Delta t^2, \Delta t\Delta x, \Delta t\Delta x^2), \end{aligned} \tag{18}$$

thus it is known that equation (13) is consistent with (2) and the overall accuracy of the method is of first order accuracy.

Consider the case: $f = const, g = 0$. Let v and w be, respectively, the accurate solution, and the numerical approximation to (2). Since both equations (12) and (13) now become (7) and (8), and in view of (14), it gives that

$$\begin{aligned} \mid v_i^{n+1} - w_i^{n+1} \mid &= \mid v_i^{n+1} - P(x_i - f_i^n\Delta t) \mid \\ &= \mid v_i^{n+1} - v^n(x_i - f_i^n\Delta t) + O(\Delta x^4) \mid \\ &\le \mid v_i^{n+1} - v^n(x_i - f_i^n\Delta t) \mid + \mid O(\Delta x^4) \mid \\ &\le \mid O(\Delta x^4) \mid . \end{aligned} \tag{19}$$

Similarly, we may also derive that

$$\mid v_{x_i}^{n+1} - w_{x_i}^{n+1} \mid \le \mid O(\Delta x^3) \mid . \tag{20}$$

Accordingly, equations (12) and (13) are of third and second order accuracy, respectively. In this case, numerical error comes only from the above interpolation.

4 WRONG SOLUTION AND ARTIFICIAL VISCOSITY

It is indicated in Sec.3 that the diffusion introduced by the operator L_2 is small. Actually being less diffusive is a feature of the CIP method [3,4]. However, it is well known that a conservative scheme with small numerical diffusion may give nonphysical solutions that violate the entropy conditions. Due to the smallness of its numerical diffusion, the CIP method may likewise endow some wrong solutions. This can be seen in the calculations of the following initial value problem

$$\begin{aligned} &v_t + (v - 3\sqrt{3}v^2(1-v)^2)_x = 0, \\ &v\mid_{t=0} = \begin{cases} 1, & x < 0, \\ 0, & x \ge 0, \end{cases} \end{aligned} \tag{21}$$

whose exact solution is

$$v = \begin{cases} 1, x \le t, \\ 0, x > t. \end{cases} \tag{22}$$

Equation (12) gives a wrong solution as depicted in Fig.1(a), where $CFL = \mid f_i^n \mid \Delta t/\Delta x$. The numerical solution seems to be stationary as

$$w_i^n = w_i^0 \tag{23}$$

with some oscillations. In this case, Lax-Wendroff scheme also gives a similar wrong solution.

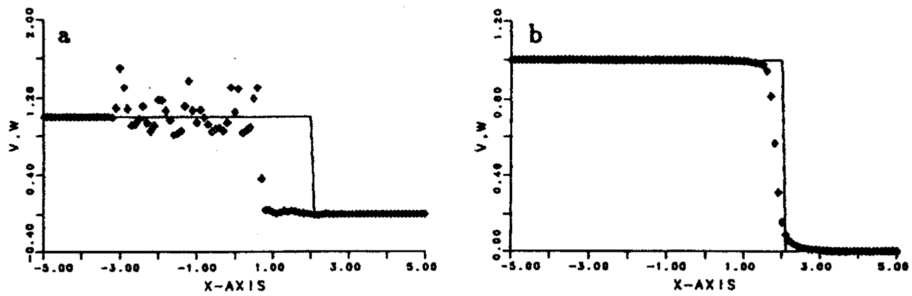

Fig.1: Initial value problem (21), t=2, CFL=0.3, $\Delta x = 0.1$.
—— exact solutions, ◇ numerical solutions,
a) CIP0 scheme. b) CIP0 scheme with (24), $\beta = 3$.

The above mentioned wrong solutions can be prevented by introducing certain form of artificial viscosity. Such artificial viscous terms have been proposed for special equations [4,5]. We now introduce an artificial viscosity to operator L_2 by adding the following two terms

$$\beta\frac{\Delta t^2}{\Delta x^2}(w_{i+1}^* + w_{i-1}^* - 2w_i^*), \quad \beta\frac{\Delta t^2}{\Delta x^2}(w_{x i+1}^* + w_{x i-1}^* - 2w_{x i}^*), \tag{24}$$

to the RHS of (7) and (8), respectively, where $\beta = const > 0$. Equation (24) can be applied to general hyperbolic equations. The calculation in Fig.1(b) shows that equation (24) can eliminate the occurrence of the above wrong solution. Numerical calculations show that equation (13) gives a wrong solution similar to that given in Fig.1(a), which can be prevented using (24). It can be verified that equation (24) actually adds the following two artificial viscous terms

$$\beta\Delta t w_{xx}, \qquad \beta\Delta t w_{xxx}. \tag{25}$$

to the RHS of (17) and (18), respectively.

Figure 2 presents the numerical solutions obtained by the CIP0 scheme for a problem of linear wave propagation ($f = 1$ and $g = 0$). Figure 3 shows those obtained with the artificial terms (24). Comparing Figs.2 and 3, we see that equation (24) does possess the capability of suppressing the numerical oscillations at edges of the square and triangle waves appearing in Fig.2, but it smears the peak value.

Equation (24) is also helpful for the stability of the CIP method. Consider the case that $f = a = const < 0$ and $g = 0$. Then equation (12) or (13) may be written as

$$W_i^{n+1} = AW_i^n, \tag{26}$$

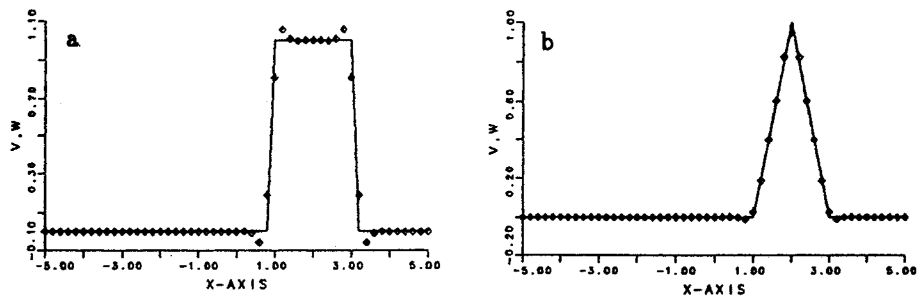

Fig.2: Linear wave propagation obtained by the CIP0 scheme,
—— exact solutions, $\diamond$ numerical solutions, $\Delta x = 0.2$, n=20, CFL=0.5.

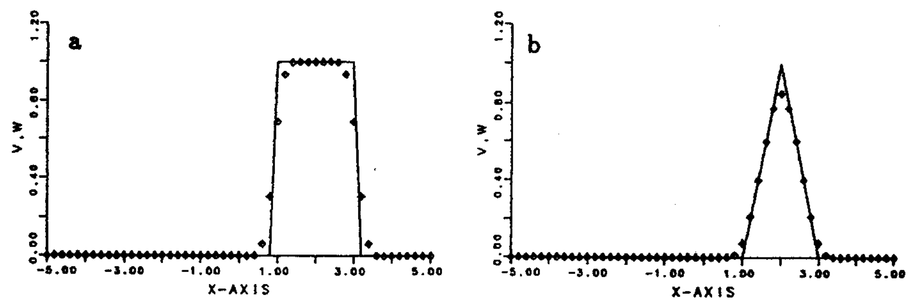

Fig.3: Linear wave propagation obtained by the CIP0 scheme with artificial terms (24),
—— exact solutions, $\diamond$ numerical solutions,
$\beta = 0.1$, $\Delta x = 0.2$, n=20, CFL=0.5.

A being a 2×2 matrix:

$$\begin{aligned} a_{11} &= 2\alpha^3(1 - S_+) + 3\alpha^2(S_+ - 1) + 1, \\ a_{12} &= \alpha^3(1 + S_+)\Delta x - \alpha^2(2 + S_+)\Delta x + \alpha\Delta x, \\ a_{21} &= \frac{6\alpha^2(1 - S_+)}{\Delta x} + \frac{6\alpha(S_+ - 1)}{\Delta x}, \\ a_{22} &= 3\alpha(1 + S_+) - 2\alpha(2 + S_+) + 1, \end{aligned} \tag{27}$$

where $\alpha = -a\Delta t/\Delta x$, S_+ is a shift operator: $S_+ w_i = w_{i+1}$. Making Fourier transformation for (26), one has the amplification matrix $\overline{A}$ as

$$\begin{aligned} \overline{a_{11}} &= 2\alpha^3(1 - e^{-j\zeta}) + 3\alpha^2(e^{-j\zeta} - 1) + 1, \\ \overline{a_{12}} &= \alpha^3(1 + e^{-j\zeta})\Delta x - \alpha^2(2 + e^{-j\zeta})\Delta x + \alpha\Delta x, \\ \overline{a_{21}} &= \frac{6\alpha^2(1 - e^{-j\zeta})}{\Delta x} + \frac{6\alpha(e^{-j\zeta} - 1)}{\Delta x}, \\ \overline{a_{22}} &= 3\alpha(1 + e^{-j\zeta}) - 2\alpha(2 + e^{-j\zeta}) + 1, \end{aligned} \tag{28}$$

in which $j=\sqrt{-1}, 0 \le \zeta \le 2\pi$. Let the eigenvalue of $\overline{A}$ be λ, then

$$\lambda = 1 - \alpha M^{\pm} e^{-j\theta}, \tag{29}$$

where $0 \le M_{\pm} \le M = const$, and

$$\begin{aligned} &\alpha M^{\pm} e^{-j\theta\pm} = \frac{-b \pm \sqrt{b^2-4c}}{2}, \\ &b = 2\alpha^3 - 4\alpha - (2\alpha^3 - 6\alpha^2 + 2\alpha)e^{-j\zeta}, \\ &c = \alpha^4 - 6\alpha^3 + 6\alpha^2 - (2\alpha^4 - 6\alpha^3 + 2\alpha^2)e^{-j\zeta} + \alpha^4 e^{-j\zeta}, \\ &b^2 - 4c = 4\alpha^6 - 20\alpha^4 + 24\alpha^3 - 8\alpha^2 + (-8\alpha^6 + 24\alpha^5 + 16\alpha^4 - 72\alpha^3 + 40\alpha^2)e^{-j\zeta} \\ &\qquad + (4\alpha^6 - 24\alpha^5 + 40\alpha^4 - 24\alpha^3 + 4\alpha^2)e^{-2j\zeta}. \end{aligned} \tag{30}$$

Equation (29) yields:

$$|\lambda| = \sqrt{1 - 2\alpha M^{\pm}\cos\theta^{\pm} + \alpha^2 M^{\pm 2}}, \tag{31}$$

Since $|\lambda| < 1$ is sufficient for the stability of (26), reducing the value of $|\lambda|$ can improve its stability. By adding (24) into (7) and (8), the eigenvalue of the above amplification matrix $\overline{A}$ now becomes

$$|\lambda_\beta| = \sqrt{|\lambda|^2 + \lambda'}, \tag{32}$$

where

$$\lambda' = \frac{4\alpha^4\beta^2}{a^4}(1-\cos\zeta)^2 + \frac{4\alpha^3\beta}{a^2}M^{\pm}(1-\cos\zeta)\cos\theta - \frac{4\alpha^2\beta}{a^2}(1-\cos\zeta). \tag{33}$$

It is easy to verify that when $\alpha \le 1/M$, or time step is sufficiently small,

$$\lambda' \le 0. \tag{34}$$

Accordingly

$$|\lambda_\beta| \le |\lambda|. \tag{35}$$

This suggests that equation (24) improves the stability of the CIP method.

5 MONOTONICITY PRESERVING CIP SCHEMES

Monotonicity preserving is useful in obtaining better resolution on discontinuity and suppressing numerical oscillations as well. The original CIP schemes do not necessarily have this property. However, the following theorem shows that some modifications can make the schemes to be monotonicity preserving.

Theorem If $P(x)$ is piecewise monotone, then (7) is monotonicity preserving under the CFL-like restriction

$$\left| \frac{f_i^n \Delta t}{\Delta x} \right| \le \frac{1}{2}. \tag{35}$$

Proof According to (35), one has

$$\begin{aligned} f_{i+1}^n \Delta t &\le \tfrac{1}{2}(x_{i+1} - x_i), \\ -f_i^n \Delta t &\le \tfrac{1}{2}(x_{i+1} - x_i), \end{aligned} \tag{36}$$

combining the two inequalities yields

$$x_i - f_i^n \Delta t \le x_{i+1} - f_{i+1}^n \Delta t, \tag{37}$$

Let $w_i^* \le w_{i+1}^*$. In view of (7) and the monotonicity of $P(x)$, it is derived that

$$\begin{aligned} w_i^{n+1} &= P(x_i - f_i^n \Delta t) \\ &\le P(x_{i+1} - f_{i+1}^n \Delta t) = w_{i+1}^{n+1}. \end{aligned} \tag{38}$$

Similarly, let $w_i^* > w_{i+1}^*$, one shall have

$$w_i^{n+1} > w_i^{n+1}, \tag{39}$$

which completes the proof of the theorem.

What is left for preserving monotonicity is to construct piecewise monotone cubical Hermite interpolation. Fortunately, there have been many good results in this aspect, such as those by Boor [10] and Huynh [11]. We now use some of the results given in [11] to construct two monotonicity preserving the CIP schemes as follows.

A monotone restriction called MP (Monotonicity Preserving) algorithm is [11]

$$\begin{aligned} w_{xi} &= minmod(w_{xi}, 3s_i), \\ s_i &= minmod(s_{i-1/2}, s_{i+1/2}), \\ s_{i+1/2} &= (w_{xi+1} - w_{xi})/\Delta x, \end{aligned} \tag{40}$$

in which

$$minmod(x, y) = \begin{cases} sgn(x) min(\mid x \mid, \mid y \mid), xy > 0, \\ 0, \qquad\qquad\qquad xy \le 0. \end{cases} \tag{41}$$

If equation (9) satisfies (40), it is a monotonous cubic Hermite interpolation. However, this is not usually true. For the monotonicity, we may use (40) to modify w_{xi}^n given by (9), thus w_i^n will be monotone within each grid cell $[x_i, x_{i+1}]$. After the modification it is known by the above theorem that equation (7) will become piecewise monotone. Accordingly, equations (7), (8), and (40) are monotonicity preserving schemes, and we call it CIPM1 scheme. Figure 4(a) shows the calculation of the linear wave propagation problem obtained by the CIPM1 scheme. It is seen that the numerical oscillations in Fig.2(a) are removed by the monotonicity. But, as indicated in [11], MP algorithm will lead 'clipping phenomenon', i.e., it causes the accuracy of (9) to degenerate at local extrema. This is seen in Fig.4(b).

In order to maintain the accuracy at local extrema, we may adopt the following uniformly high order accurate interpolation given by Huynh [11]

$$w_{xi} = minmod(w_{xi}, t_{max}), \tag{42a}$$

where

$$\begin{aligned} t_{max} &= sgn(t_i) max(3 \mid s_i \mid, 3 \mid t_i \mid /2), \\ t_i &= minmod(R_{xi-1/2}(x), R_{xi+1/2}(x)), \\ R_{xi+1/2}(x) &= s_{i-1/2} + d_{i+1/2}\Delta x, \\ R_{xi-1/2}(x) &= s_{i-1/2} - d_{i+1/2}\Delta x, \\ d_{i+1/2} &= minmod(d_i, d_{i+1}), \\ d_i &= (s_{i+1/2} - s_{i-1/2})/2\Delta x. \end{aligned} \tag{42b}$$

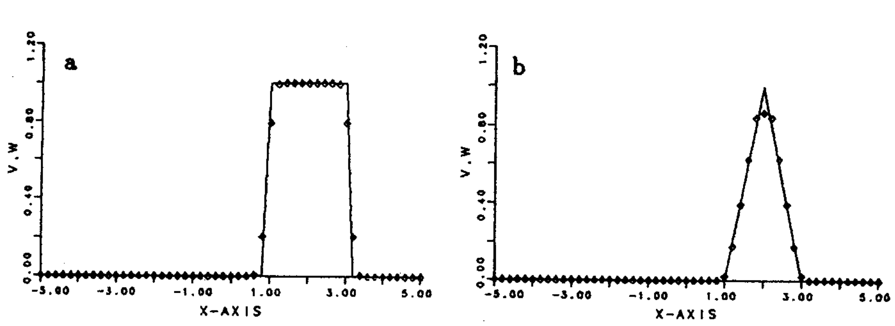

Fig.4: Linear wave propagation obtained by the CIPM1 scheme,
—— exact solutions, ◇ numerical solutions, $\Delta x = 0.2$, n=20, CFL=0.5.

Algorithm (42) is monotone and stable. If (42) is used to modify w_{xi}^n given by (9), $P(x)$ will be piecewise monotone, and, when $v \in C^3$ and w_i and w_{xi} are, respectively, of third and second order accuracy. Furthermore, $P(x)$ and $P_x(x)$ will be, respectively, of uniformly third and second order accuracy. Equations (7), (8) and (42) are referred to as the CIPM2 scheme, and they are monotonicity preserving with uniformly high order accuracy. The calculations of above linear wave propagation problem are given in Fig.5. Comparing Figs.2, 4 and 5, it is seen that the CIPM2 not only removes the oscillations, but also recovers the accuracy at the extremum.

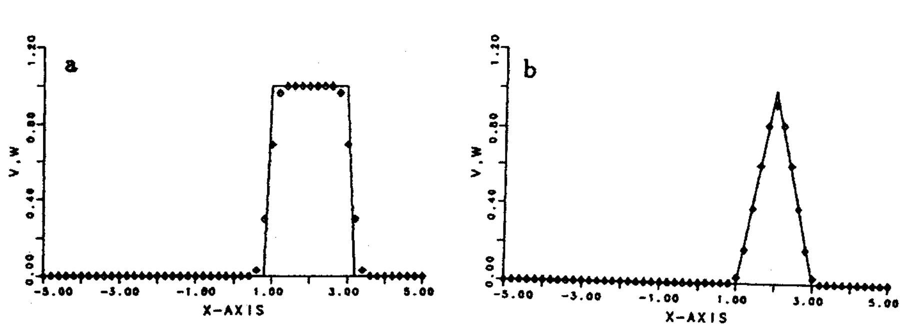

Fig.5: Linear wave propagation obtained by the CIPM2 scheme,
—— exact solutions, ◇ numerical solutions, $\Delta x = 0.1$, n=20, CFL=0.5.

Compared with the CIP0, the schemes CIPM1 and CIPM2 are more CPU time consuming due to the constraints (40) and (42). However, some simpler constraints may be employed to modify the CIP0 scheme. For more details, *c. f.* [7].

6 COMPUTATIONS OF THE EULER EQUATIONS

In this section we apply artificial terms (24) to computations of the one dimensional Euler equations, which may be written as

$$V_t + uV_x = H, \tag{43a}$$

where

$$V = (\rho, u, e)^T, \quad H = (-\rho u_x, p_x, -\frac{p}{\rho}u_x)^T. \tag{43b}$$

Here ρ, u, and e are density, velocity, and internal energy, respectively, and $e = p/(\gamma - 1)$, γ the ratio of specific heats. We will show calculations of a shock tube problem that is the test problem in [3]. In our calculations (12) is used and (5) reads

$$\begin{aligned} \rho_i^* &= \rho_i^n - \frac{\Delta t}{2\Delta x}\rho_i^n(u_{i+1}^n - u_{i-1}^n), \\ u_i^* &= u_i^n - \frac{\Delta t}{2\Delta x}\frac{p_{i+1}^n - p_{i-1}^n}{\rho_i^n}, \\ e_i^* &= e_i^n - \frac{\Delta t}{4\Delta x}\frac{p_i^n(u_{i+1}^* - u_{i-1}^* + u_{i+1}^n - u_{i-1}^n)}{\rho_i^n}. \end{aligned} \tag{44}$$

The computation obtained with no artificial viscous terms is given in Fig.6(a). Compared with the exact solution, it is seen that the numerical solution is wrong since it presents no shock moving to the right. By adding the artificial terms (24) to the CIP0 scheme, the shock is resolved as shown in Fig.6(b). In the calculations of the same problem, the authors of [3] add the following artificial viscous term to the pressure term:

$$q_i = \begin{cases} d(-\sqrt{\gamma\rho_i p_i}(u_{i+1} - u_{i-1}) + \frac{\gamma+1}{2}\rho_i(u_{i+1} - u_{i-1})^2), & u_{i+1} - u_{i-1} \le 0, \\ 0, & u_{i+1} - u_{i-1} > 0. \end{cases} \tag{45}$$

Numerical solution with this term is depicted in Fig.6(c), and it has some oscillations behind the shock. However, if (24) is employed, these oscillations are suppressed (Fig.6(d)).

7 CONCLUDING REMARKS

In the current paper, it is shown that the CIP0 scheme is consistent. In linear and homogeneous cases, its accuracy relates only to the accuracy of the interpolation. It is also indicated that due to the smallness of its numerical viscosity, the CIP method may give wrong solutions. The artificial viscous terms introduced in this paper are effective for avoiding the wrong solutions, and they can be used in general hyperbolic equations.

The monotonicity preserving CIP schemes proposed in this paper, which can be of uniformly high order accuracy, serve the purpose of obtaining better resolution for discontinuities. Like the CIP1 and CIP2 schemes developed in [3] for the same purpose, which modify the derivatives over the space at each grid node. Our schemes, on the other hand, need not track the discontinuities, which is necessarily in the CIP1 and CIP2 schemes, thus they are simpler in this sense. It would be useful to apply the monotonicity preserving CIP schemes to calculations of practical flow problems.

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CONTENTS