Proceedings of
The Seventh International Symposium on Computational Fluid Dynamics

September 15 - 19, 1997
Beijing, China

Edited by F. G. ZHUANG

International Academic Publishers

WHY NONCONSERVATIVE INTERFACE ALGORITHMS MAY BE APPLICABLE: ANALYSIS FOR CHIMERA GRIDS [1]

Tang Hansong
(School of Space Technology, Beijing University of Aeronautics and Astronautics, Beijing, China, 100083)
Zhou Tie
(School of Mathematics, Peking University, Beijing, China, 100871)

Abstract

In computations of flowfields with discontinuities by zonal methods, it is believed that conservation at grid interfaces is necessary to guarantee the correct discontinuity locations and jumps. In this paper we investigate the calculations of 3D systems of conservation laws using nonconservative treatments at interfaces between Chimera embedding grids. Under some assumptions, we prove that conservation error due to nonconservative interface schemes is bounded and tends to zero as mesh size approaches zero. We also show the limit of the solutions obtained by such schemes are weak solutions of the systems. A numerical example is presented to illustrate these conclusions.

Introduction

In simulating flows over complex geometries by zonal methods, it is crucial to choose proper algorithms for grid interfaces. Grid interface algorithms fall into two categories: conservative treatments and nonconservative treatments[1].

A conservative treatment ensures conservation of mass, momentum, and energy at grid interfaces in calculations of Euler equations, and, if a numerical solution converges, it guarantees that the solution converges to a weak solution of the equations. As a result, conservation at the interfaces is now widely claimed and its procedure has been studied by many authors, cf. [2,3,4]. Nevertheless, there are some negative aspects about them, cf. [5,6]. Most important of all, implementing the conservation is very complicated, if not impossible, in 2- and 3-dimensional Chimera embedding grids, which are often encountered in calculations with complex geometries.

There have been systematic studies about nonconservative treatments for grid interfaces as well as many papers reporting their successful applications, cf. [7,8]. However, it is believed that a conservative treatment between grids is necessary to ensure correct discontinuity locations and jumps, and numerical examples do show that a nonconservative treatment may cause inaccurate shock jumps and shock locations, especially when zonal boundaries are close to the exact shock locations (see [9]). As indicated in [4], it is unclear about what the effects of a nonconservative interface condition would be on the overall accuracy and correctness of numerical solutions.

Therefore, it is natural to ask why nonconservative treatments may be used and whether they can provide right jumps and locations of discontinuities. The authors have investigated nonconservative

[1] Sponsored by the Chinese Natural Science Fundation

conditions at grid interfaces and given explanations to these questions[10]. In the current paper, we present the explanations via two theorems and illustrate them with a numerical example.

Governing Equations and Their Discretization

Consider the calculation of the initial value problem of a 3D system of conservation laws

$$\begin{aligned} &U_t(x,y,z,t) + F_x(U(x,y,z,t)) + G_y(U(x,y,z,t)) + H_z(U(x,y,z,t)) = 0, \\ &U(x,y,z,0) = U_0(x,y,z). \end{aligned} \tag{1}$$

Here, $t > 0, x, y, z \in \Omega$, $\Omega = \langle x, y, z \mid x, y, z = -\infty, +\infty \rangle$, $U_0(x,y,z)$ has a compact support, and, for convenience, it is assumed that $F(0) = G(0) = H(0) = 0$. Let Ω be divided into Ω_A and Ω_B by a zonal interface Σ (Fig. 1). In Ω_A the system is written in coordinate (x,y,z,t), while in Ω_B it is written in coordinate (ξ,η,ς,t),

$$\overline{U}_t(\xi,\eta,\varsigma,t) + \overline{F}_\xi(\overline{U}(\xi,\eta,\varsigma,t)) + \overline{G}_\eta(\overline{U}(\xi,\eta,\varsigma,t)) + \overline{H}_\varsigma(\overline{U}(\xi,\eta,\varsigma,t)) = 0. \tag{2}$$

On Σ $\xi = const$.

As depicted in Fig. 1, the calculation on Ω_A and Ω_B is done on grid G_A and G_B, respectively, which are comprised of the lines $x, y, z = const$ and lines $\xi, \eta, \varsigma = const$, respectively. The two grids overlap in an arbitrary way, and they are so fine that Σ is nearly a plane within each cell of G_A. Let $\{i_B, j_B, k_B\}$ be nodes of G_B involved in the calculation on Ω_B, and $\{i_{BI}, j_{BI}, k_{BI}\}$ be its nodes located on Σ. In order to obtain the solutions at the nodes of G_A that face and are next to Σ (represented by circles in Fig. 1), conditions are needed at some of its nodes in Ω_B (marked with dots in Fig. 1). For each j, k there is a such node $(i_A(j,k), j, k)$, for each i, k a node $(i, j_A(i,k), k)$, and for each i, j a node $(i, j, k_A(i,j))$. These nodes make up $\{i_{AI}, j_{AI}, k_{AI}\}$. Let all the nodes of G_A involved in the calculation on Ω_A be $\{i_A, j_A, k_A\}$. Furthermore, let Σ_A be a surface comprised by many small surfaces

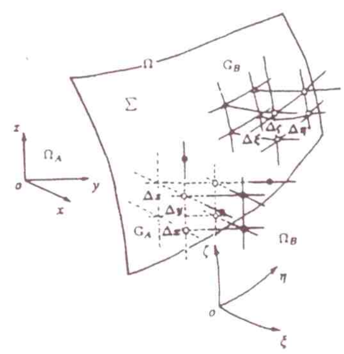

Fig. 1 Ω_A and Ω_B and their grids

$$\begin{aligned} &\langle x,y,z \mid x = x_{i_A(j,k)-1/2,j,k},\, y,z \in (y_{i_A(j,k),j-1/2,k}, y_{i_A(j,k),j+1/2,k}] \times (z_{i_A(j,k),j,k-1/2}, z_{i_A(j,k),j,k+1/2}]\rangle, \\ &\langle x,y,z \mid y = y_{i,j_A(i,k)-1/2,k},\, x,z \in (x_{i-1/2,j_A(i,k),k}, x_{i+1/2,j_A(i,k),k}] \times (z_{i,j_A(i,k),k-1/2}, z_{i,j_A(i,k),k+1/2}]\rangle, \\ &\langle x,y,z \mid z = z_{i,j,k_A(i,j)-1/2},\, x,y \in (x_{i-1/2,j,k_A(i,j)}, x_{i+1/2,j,k_A(i,j)}] \times (y_{i,j-1/2,k_A(i,j)}, y_{i,j+1/2,k_A(i,j)}]\rangle, \end{aligned} \tag{3}$$

and Σ_B be the surface $\xi = 1/2$. It is readily seen that when mesh size goes to zero, Σ_A will overlap with Σ.

We define that $\Omega^A_{i,j,k} = \langle x,y,z \mid x,y,z \in (x_{i-1/2}, x_{i+1/2}] \times (y_{j-1/2}, y_{j+1/2}] \times (z_{k-1/2}, z_{k+1/2}]\rangle$, $i,j,k \subset \{i_A, j_A, k_A\}$, $\Omega^B_{i,j,k} = \langle \xi,\eta,\varsigma \mid \xi,\eta,\varsigma \in (\xi_{i-1/2}, \xi_{i+1/2}] \times (\eta_{j-1/2}, \eta_{j+1/2}] \times (\varsigma_{k-1/2}, \varsigma_{k+1/2}]\rangle$, $i,j,k \subset \{i_B, j_B, k_B\}$, $\Omega_{AD} = \bigcup \Omega^A_{i,j,k}$, $i,j,k \subset \{i_A, j_A, k_A\} \setminus \{i_{AI}, j_{AI}, k_{AI}\}$, $\Omega_{BD} = \bigcup \Omega^B_{i,j,k}$, $i,j,k \subset \{i_B, j_B, k_B\} \setminus \{i_{BI}, j_{BI}, k_{BI}\}$, $\Omega_I = \Omega \setminus \Omega_{AD} \setminus \Omega_{BD}$. Then, based on some volume-wise constant functions, on G_A and G_B the formulation of explicit conservative schemes with a 3 point stencil can be written as, respectively,

$$\begin{aligned} U^A_\Delta(x,y,t+\Delta t) \;=\; U^A_\Delta(x,y,z,t) &- \frac{\Delta t}{\Delta x}(F_\Delta(x+\Delta x/2,y,z,t) - F_\Delta(x-\Delta x/2,y,z,t)) \\ &- \frac{\Delta t}{\Delta y}(G_\Delta(x,y+\Delta y/2,z,t) - G_\Delta(x,y-\Delta y/2,z,t)) \\ &- \frac{\Delta t}{\Delta z}(H_\Delta(x,y,z+\Delta z/2,t) - H_\Delta(x,y,z-\Delta z/2,t)), \end{aligned} \tag{4}$$